SOME EXTENSIONS OF MULTIREGIONAL INPUT-OUTPUT ANALYSIS

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ABSTRACT Originally, applications of input-output models were carried out at national levels. More recently, interest in economic analysis at the regional level has led to modifications of input-output models in order to deal with regional issues. The main theme of this paper lies in showing how to extend and elaborate conventional input-output analysis at regional level to derive some meaningful policy recommendations from a practical point of view. Judging from recently developed analytical tools such as internal and/or external multiplier matrices, augmented input coefficient matrices, hierarchical feedback loops and interregional linkages including positive and/or negative feedback effects, there is a need for theoretical assessments need to be developed alongside empirical applications.

1. INTRODUCTION

Spatial Interactions (or interregional relationships) are the main subject of this paper. From the methodological viewpoint, the paper can be divided into the following two parts: the definition of Spatial Interactions - which might be called the “diagnosis”, is necessary for a good understanding of the region of study. The second part is what might be called the “therapy”, dealing with deriving sound policy recommendations from the preceding empirical studies.

The paper is organised as follows: in Section 2 an examination of the use of the conventional input-output method is provided, together with some references to recently developed analytical tools. A brief description of data-availability follows in Section 3. Section 4 describes an empirical application of the method, to certain parts of Japan’s interregional system. Concluding remarks complete the paper in Section 5.

2. ANALYTICAL METHOD

2.1 Why Use the Input-Output Model?

The Input-Output Model is a most fundamental and useful framework, used not only as a descriptive device but also as an analytical tool. Formally speaking, it consists of the following three matrices: 1) the Input-Output Table (or Transaction Matrix, i.e., $X = [x_{ij}]$ and/or Social Accounting Matrix, i.e., $SAM$), which might be regarded as a descriptive device. 2) the Input Coefficient Matrix, i.e., $A = [a_{ij}]$ and 3) the Leontief Inverse Matrix, i.e., $B = (I - A)^{-1}$, which might be regarded as an analytical tool. In order to convert $X$ to $B$, the following three technical assumptions
are usually introduced:
1. Constant Returns to Scale
2. Convexity of the Isoquant Surfaces
3. Fixed Coefficients of Production.

The first assumption signifies that each production function has the property of first order homogeneity. Mathematically, it can be written as follows:

$$X_i = \min \left( \frac{x_{1i}}{a_{1i}}, \frac{x_{2i}}{a_{2i}}, \ldots, \frac{x_{ni}}{a_{ni}}, \frac{v_{oi}}{a_{oi}} \right) ; \quad i = 1, 2, \ldots, n$$

where $X_i$ is the gross output of sector $i$,
- $x_{ji}$ is the intermediate input of sector $j$ ($j=1,2,\ldots,n$) by sector $i$,
- $a_{ji}$ is the input coefficient of sector $j$ ($j=1,2,\ldots,n,0$) in order to produce one unit of output in sector $i$,
- $V_{oi}$ is the primary input of value-added (i.e., sector $j=0$) by sector $i$.

The second assumption tells us that, theoretically, the generalised law of decreasing returns always holds in input-output models. Hence, if we denote the elasticity of substitution between each intermediate input by $\sigma$ then the assumption can be explained as follows:

$$\sigma = \frac{d}{d \log \left( \frac{MP_{ik}}{MP_{jk}} \right)} \frac{MP_{ik}}{MP_{jk}} = \frac{d}{d \log \left( \frac{MP_{ik}}{MP_{jk}} \right)} \frac{x_{jk}}{x_{ik}} = 0$$

where $MP_{ik}$ and $MP_{jk}$ stand for the marginal productivity of intermediate inputs $i$ and $j$ by sector $k$, respectively.

The third assumption signifies that the input coefficients of each sector, which can be derived from the following operation, are always constant over time, regardless of the input scale:

$$A = [a_{ij}] = \begin{bmatrix} \frac{x_{ij}}{X_j} \end{bmatrix} ; \quad i, j = 1, 2, \ldots, n$$

where $a_{ij}$ is the input coefficient of sector $i$ in order to produce one unit of output in sector $j$ ($i,j=1,2,\ldots,n$),
- $x_{ij}$ is the intermediate input of sector $i$ by sector $j$ ($i,j=1,2,\ldots,n$),
- $X_j$ is the gross output of sector $j$.

Criticism has usually centered around assumption 3 - Fixed Coefficients of Production. Naturally, various viewpoints have emerged. The different viewpoints can be classified as follows:
1. Those who assert that the assumption is theoretically dubious, but that as the first approximation to analysis, its adoption might be permissible.
2. Those who assert that the assumption has been empirically as well as statistically verified, hence the assumption might be approved.
3. Those who assert that the assumption has been verified by the theorem on substitution, etc., hence the assumption should be positively adopted.

2.2 Qualifications and Limitations of Input-Output Model

An input-output model must be qualified for its operationality and/or manipulation with quantitative measures. Theoretically speaking, the product-determining mechanism, specified as \( X = (I - A)^{-1} F \), is independent from the price-determining mechanism, specified as \( P = (I - A)^{-1} V \), and vice versa, where \( X \) and \( F \) are vectors of output and final demand respectively, and \( P \) and \( V \) are vectors of price and value-added respectively.

The input-output model can be easily extended to the regional level. Three different types of regional models have been proposed and empirically applied. The Isard-type model is the most primitive and fundamental and can be derived by defining the interregional input coefficients directly. By separating the input coefficients from the trade coefficients we can derive the Chenery-Moses-type model, which can be regarded as a modification of the Isard-type model. The Leontief balanced model differs from the former two models by classifying the goods under study into three categories - ‘national goods’, ‘regional goods’ and ‘local goods’. More detailed information and additional explanations are given in Miller and Blair (1985) and Ihara (1997).

A great advantage of the input-output model is that it covers the range between extreme aggregation and complete disaggregation. Another major advantage lies in its stress on interdependence; it is the only branch of economic theory which shows empirically how “everything depends upon everything else”. In short, it has brought to realization, in an operational form, the grand design of General Equilibrium Theory, which had its roots in the work of Quesnay and Walras.

However, we must recognize the gap between the theoretical and the empirical. In order to clarify this point, consider the regional input-output models for Japan. The Ministry of International Trade and Industry’s published input-output tables for the years 1960, 1965, and 1970 onwards are based on the Isard-type model. In the latest table (1990), the regional classification is based on a division of the country into 9 regions, such as Kinki, Chugoku and Shikoku.

These tables can be converted into flexible analytical tools by introducing the assumptions noted in Section 2.1, making them as detailed or as condensed as necessary for any given purpose. How “open” or “closed” an input-output table should be depends largely upon the purpose for which it is to be used. It should be noted however, that tracing a set of transactions to complete a comprehensive table is a painstaking task that can consume time and money in equally large proportions. In addition, to make matters worse, those completed tables still remain as a “special” or “partial” model within the framework of General Equilibrium Theory. Due to these factors, input-output tables have not always been fully utilized for economic forecasting and/or policy formation. Recently, new methods have been proposed
which offer the promise of narrowing the gap between theoretical justification and empirical implementation (see, for example, Shoven and Whalley (1992), Miyagi (1997), Ihara (1997), Hewings and Madden (1995), Sonis and Hewings (1997) and Sonis, Hewings, Guo and Hulu (1997)).

3. DATA AVAILABILITY

3.1 Outline of the Study Regions

It should be noted that regional self-autonomy is highly dependent on the size of the region (or the degree of “openness” of the study). In order to examine the “Spatial Interactions” (or interregional relationships) empirically, we shall consider the Shikoku region of Japan and its surrounding regions, Chugoku and Kinki. The main reasons for selecting those three regions are as follows: Shikoku is geographically isolated from the mainland. It has also long been an economically stagnant region. Recently, the transport-infrastructure, which includes the Honshu-Shikoku connecting bridges as well as the Highway network system, has been much improved. Due to this, it is now necessary to measure the spatial dependency of the Shikoku region on other regions, such as the Chugoku (with Hiroshima as its center) and Kinki regions (with Osaka as its center), in order to make sound policy recommendations.

A brief outline of each regional economy is given in Table 1. From this table it can be seen that the population of the Kinki region is five times that of the Shikoku region and that the Gross Regional Product of Kinki is six times that of the Shikoku region. There is every likelihood that the difference will be maintained or increased in the near future.

3.2 Available Data

Generally speaking, data and/or statistics, which show spatial interactions (or interregional relationships), are unavailable compared with intraregional data and/or statistics. However for our purposes, as long as we take account of the impact of the transport-infrastructure on the regional economy, more attention should be paid to

<table>
<thead>
<tr>
<th></th>
<th>Area</th>
<th>Population</th>
<th>Gross Regional Product</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Km²</td>
<td>Percent</td>
<td>Thousand</td>
</tr>
<tr>
<td>Shikoku</td>
<td>18,784</td>
<td>(100)</td>
<td>4,221</td>
</tr>
<tr>
<td>Chugoku</td>
<td>31,805</td>
<td>(169)</td>
<td>7,764</td>
</tr>
<tr>
<td>Kinki</td>
<td>31,346</td>
<td>(167)</td>
<td>21,210</td>
</tr>
</tbody>
</table>

The figures in parentheses are the relative shares derived from taking Shikoku as the base. Population as of October 1, 1990. Gross Regional Product as of 1990 fiscal year.
the changes in *interregional* relationships in addition to the changes in *intraregional* relationships. In this respect, the following three types of different data are available with respect to our study regions:

1. Road Traffic Census
2. Resident Basic Register
3. Interregional Input-Output Table.

The Ministry of Construction and the related administrative divisions undertake Road Traffic Censuses, in order to obtain statistics about the real state of road traffic. Data is assembled by roadside interview of owners on their point of origin and destination. It should be noted that the data are all measured in physical units such as weights and/or number of persons. The available data on O-D pairs on weekdays are for 1985, 1990 and 1994, while the data on weekends and holidays are for 1990 and 1994.

The Statistics Department of the General Affairs Agency assembles the Resident Basic Register every year on 1st October. Hence, annual data is available for empirical investigations. Its primary purpose is to clarify the demographic changes, or the circumstances of inflow and outflow of residents among Japan’s different administrative divisions (prefectures, for example). The data are measured in physical units of persons. The Interregional Input-Output Tables are compiled by the Ministry of International Trade and Industry so as to reveal the interregional economic relationships. The great advantage of this data is that they are measured in monetary units. As a result, we can make a more detailed economic evaluation of interregional relationships. The tables do have some drawbacks. The Interregional Input-Output Tables are published only every five years, (starting in 1960) due to the time and expense involved in compiling them. The latest Interregional Input-Output Tables are for 1990, which means time-lag problems when the tables are used for economic forecasting. Additionally, the regional breakdown in the tables seems to be too highly aggregated when compared with the other two types of available data. For example, the Shikoku, Chugoku and Kinki regions are treated as one, and hence the interprefectural relationships within those regions cannot be identified. For reference, Shikoku region consists of the prefectures of Tokushima, Kagawa, Ehime and Kochi, while Chugoku region consists of the prefectures of Tottori, Shimane, Okayama, Hiroshima and Yamaguchi. The Kinki region has the seven prefectures of Fukui, Shiga, Kyoto, Osaka, Hyogo, Nara and Wakayama.

### 4. EMPIRICAL APPLICATION

Utilizing the available data to the full extent, we carried out some empirical investigations of the regions. The following is a brief explanation of the analytical method and our results.

#### 4.1 Analysis Utilizing the “Road Traffic Census”

In order to deal with the innumerable number of figures in the Road Traffic Census in an efficient way, we followed simple analytical methods:
Analytical Methods

The Cohesion Index (designated by $C_i$) can be defined as follows:

$$C_i = \frac{E_i}{\sum_{i=1}^{n} E_i}$$

where $C_i$ is the Cohesion Index of region $i$, $E_i$ is the $i$-th element of the $n$-dimensional row vector $E$.

More specifically, the following two kinds of indices can be defined:

$$OC_{ki} = \frac{X_{ki}}{X_k},$$

where $OC_{ki}$ is the Outflow Cohesion Index of region $k$ to region $i$, $X_{ki}$ is the number of weights and/or persons moving from region $k$ to region $i$, $X_k$ is the total number of weights and/or persons moving from region $k$.

$$IC_{ki} = \frac{X_{ki}}{X_i},$$

where $IC_{ki}$ is the Inflow Cohesion Index of region $i$ from region $k$, $X_i$ is the total number of weights and/or persons moving to region $i$.

By taking the two indices together, the following two further kinds of index can also be defined:

$$TCD_{ki} = OC_{ki} - IC_{ki} \quad (\geq 0)$$

where $TCD_{ki}$ is the Total Cohesion Index by Difference between region $k$ and region $i$.

$$TCQ_{ki} = \frac{OC_{ki}}{IC_{ki}} \quad (\geq 1)$$

where $TCQ_{ki}$ is the Total Cohesion Index by Quotient between region $k$ and region $i$.

Finally, the Distribution Coefficient Index (designated by $\mu_{ki}$) can be defined as follows:

$$\mu_{ki} = \frac{X_{ki}}{X_k X_i / X..}$$

where $\mu_{ki}$ is the Distribution Coefficient Index between region $k$ and region $i$, $X..$ is the total number of weights and/or persons in all regions.
Some Extensions of Multiregional Input-Output Analysis

Analytical Results

Our findings are summarized as follows:

1. Regarding commodity and passenger flows, the relationship between Chugoku and Kinki has intensified over time, while the relative importance of Shikoku has diminished.

2. The discrepancy between the commodity flow "Outflow Cohesion Index" ($OC_k$) and the "Inflow Cohesion Index" ($IC_k$) turned out to be much greater than that of the passenger flow.

3. The results of the "Distribution Coefficient Index" ($\mu_k$), show that the interregional friction of the commodity flow has been mitigated over time, compared to "passenger flow". Hence, the lead-lag relation might be revealed therein.

4.2 Analysis Utilizing the "Resident Basic Register"

Our main concern, in this section, is to examine whether changes in migration patterns can be related to the improvement of the transport-infrastructure, particularly the construction of the Seto-Ohashi bridge, which was completed in 1988. Therefore, we specified the following regression model using dummy variables in order to measure the impact:

Analytical Methods

\[ Y_t = (a_1 + a_2 D_t)X_t + (b_1 + b_2 D_t) + \epsilon_t \]

where \( Y_t \) is the number of moving population at year \( t \),
\( X_t \) is the explanatory variable, (say, the time variable at year \( t \)),
\( D_t \) is the dummy variable, which takes on 0 for the years of 1982 through 1988 and 1 for the years of 1989 through 1995.

Analytical Results

Our findings are summarized as follows:

1. \( a_1 \) takes a negative sign, while \( a_2 \) takes a positive one, showing that the decreasing trend of population movement, particularly after the completion of the Seto-Ohashi bridge in 1988, is now mitigated (or locked in) to some extent.

2. The trend in population movement between Shikoku and Chugoku is decreasing at a higher rate than between the rest of the regions.

3. The results suggest that the Seto-Ohashi bridge has had two types of shifting effect on movement of population between Honshu and Shikoku. One is the slope-shift effect, while the other is the intercept-shift effect; both effects are statistically significant.
4.3 Analysis by Utilizing “Interregional Input-Output Table”

In order to measure the complicated interregional feedback between different regions in a compact way, we have already proposed how to decompose the Leontief Inverse Matrix, i.e., 
\[ (I - A)^{-1} \]
into economically meaningful concepts such as the “Augmented Input Coefficient”, “External Matrix Multiplier”, etc. The details are given in Yamada and Ihara (1969). In this section, we only refer to the following concepts:

**Analytical Methods**

The Augmented Input Coefficient Matrix (designated by \( A_{ij}^k \)) can be defined as follows:

\[
A_{ij}^k = A_{ij} + A_{ik} B_{kj} A_{kj}
\]

where \( A_{ij}^k \) is the Augmented Input Coefficient Matrix of region \( j \) from region \( i \) via region \( k \),
\( A_{ij} \) is the ordinary Input Coefficient Matrix of region \( j \) from region \( i \),
\( A_{ik} \) is the ordinary Input Coefficient Matrix of region \( k \) from region \( i \),
\( B_{ik} = (I - A_{ik})^{-1} \) is the Internal Matrix Multiplier of region \( k \),
\( A_{kj} \) is the ordinary Input Coefficient Matrix of region \( j \) from region \( k \).

In addition, the following two kinds of Coefficient Matrices can also be defined in order to interpret the partial interregional repercussion effects:

\[
\alpha_{ij} = B_{ik} A_{ij} = (I - A_{ik})^{-1} A_{ij}
\]

where \( \alpha_{ij} \) is the Coefficient Matrix of Inducement to Production of region \( j \) on region \( i \).

\[
\beta_{ij} = A_{ij} B_{ij} = A_{ij} (I - A_{ij})^{-1}
\]

where \( \beta_{ij} \) is the Coefficient Matrix of Inducement to Input of region \( j \) on region \( i \).

**Analytical Results**

In our empirical studies, we focused our interest on Shikoku, Chugoku, and Kinki, giving them the suffixes 1, 2, and 3, respectively. Our results, based on the Augmented Input Coefficient Matrix of our analytical device, can be illustrated in Figure 1. With the aid of this concept, we can derive the following implications from our numerical results:

1. There exists a significant difference between the direct input coefficient matrices of Shikoku’s products in the Chugoku and Kinki regions, respectively. In other words, the former \( (A_{12}) \) has more weight compared to the latter \( (A_{13}) \). On the
other hand, the indirect input-inducing effect derived from Chugoku to Shikoku via Kinki ($A_{13}B_{32}A_{32}$) turns out to be relatively greater than that derived from Kinki to Shikoku via Chugoku ($A_{12}B_{22}A_{23}$). The former has a multiplier effect of 6.3 per cent, while the latter shows 5.0 per cent on average.

2. It may be that these different effects are brought about by the different input structure in each region. First of all, the weight of $A_{32}$ is dominant, compared with $A_{23}$. In the second place, the output-inducing effect of $B_{33}$ is larger than that of $B_{22}$ and last but not least, the weight of $A_{13}$ is much smaller than that of $A_{12}$. As a result, the totally augmented effect, through the indirect route from Chugoku to Shikoku via Kinki ($A_{13}B_{32}A_{32}$) turn out to be relatively greater than its counterpart through the indirect route from Kinki to Shikoku via Chugoku ($A_{12}B_{22}A_{23}$).

3. In addition, with regard to industries in Shikoku, we can find out some new facts. Those industries which receive a high indirect effect from Chugoku to Shikoku via Kinki ($A_{13}B_{32}A_{32}$) are Miscellaneous Manufacturing (0.5 per cent), Metal (0.2 per cent), Commerce & Transportation (0.1 per cent). On the other hand, those industries which receive a relatively higher indirect effect from Kinki to Shikoku via Chugoku ($A_{12}B_{22}A_{23}$) are Miscellaneous Manufacturing (0.2 per cent), Commerce & Transportation (0.1 per cent), and Agriculture, Forestry & Fisheries (0.1 per cent).

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**Figure 1.** Augmented Input Effects Derived through Indirect Routes Based on Table for 1990
5. CONCLUDING REMARKS

It has been often pointed out that interregional feedback effects are much smaller in the real world. In order to corroborate this assertion, Miller and Blair (1985) introduced the summarised results from several studies. Our empirical results on interregional feedback might bring about additional information to add to their chronological overviews of these results.

When defining a region, Richardson (1979) observed: “Clearly, a region cannot be treated as a closed system, for openness is its essence.....If regions are open systems, key exogenous variables must be specified more carefully, the greater possibility of disequilibrating processes must be recognized, models should be less deterministic, and regional economic projections accepted to be more uncertain”. The key property of an economic region is its degree of openness, yet in most countries interregional trade and capital flow are recorded inadequately at best, frequently not at all. In addition, as is often pointed out, regional boundaries are so open that regional income received is not the same as regional domestic product.

In order to deal quantitatively with interregional relationships and/or feedback effects, some systematic social accounts at regional level are required. In this context, there are five main types of social accounts proposed or used at the regional level:

1. Income and Product accounts.
2. Balance-of-Payments accounts.
4. Input-Output accounts.
5. Wealth accounts.

It should be noted that the last one is quite different from the rest, since it deals with stocks rather than flows. Input-Output accounts, however, provide the consistency checks to back up input-output models. It is arguable that interindustry accounts are more useful at the regional level than Income and Product accounts which deal only with final goods and services, because interindustry relations and intermediate supply of goods may be critical to a backward region’s economic performance.

In short, the “openness” of regions makes it almost impossible to prevent the destabilizing tendencies of interregional spillovers. The dominant focus of regional economic policy, however, is the pursuit of long-term strategies to improve the economic performance of backward regions and to maintain a balance between economic growth and environmental quality in prosperous regions. This requires a different framework and continuing efforts to evaluate regional economic policy.

REFERENCES

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